Trigonometric functions and hyperbolic trigonometric functions

The hyperbolic trigonometric functions are

hyperbolic sine	$\sinh x = \dfrac{e^x - e^{-x}}{2}$	hyperbolic cotangent	$\coth x = \dfrac{\cosh x}{\sinh x}$
hyperbolic cosine	$\cosh x = \dfrac{e^x + e^{-x}}{2}$	hyperbolic secant	$\operatorname{sech} x = \dfrac{1}{\cosh x}$
hyperbolic tangent	$\tanh x = \dfrac{\sinh x}{\cosh x}$	hyperbolic cosecant	$\operatorname{cosech} x = \dfrac{1}{\sinh x}$

Note that

- $\operatorname{cosec}(x)$ is sometimes written as $\csc(x)$
- $\operatorname{cosech}(x)$ is sometimes written as $\operatorname{csch}(x)$
- $\sin^{-1}(x)$ is sometimes written as $\arcsin(x)$
- $\sinh^{-1}(x)$ is sometimes written as $\operatorname{arcsinh}(x)$
- $\sin^{-1}(x)$ does *not* mean $\frac{1}{\sin x}$.

Here are some of the commonly used trigonometric identities together with hyperbolic trigonometric identities for comparison.

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$
$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$
$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$
$$\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$$
$$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\tan(-x) = -\tan x$$

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$
$$\tanh(-x) = -\tanh x$$

$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$\frac{d}{dx}\sin(x) = \cos(x)$
$\frac{d}{dx}\cos(x) = -\sin(x)$
$\frac{d}{dx}\tan(x) = \sec^2(x)$
$\frac{d}{dx}\cot(x) = -\operatorname{cosec}^2(x)$
$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
$\frac{d}{dx}\operatorname{cosec}(x) = -\cot(x)\operatorname{cosec}(x)$
$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
$\frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2}$
$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$
$\frac{d}{dx}\operatorname{cosec}^{-1}(x) = \frac{-1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}\sinh x = \cosh x$
$\frac{d}{dx}\cosh x = \sinh x$
$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$
$\frac{d}{dx}\coth x = -\operatorname{cosech}^2 x \ (x \neq 0)$
$\frac{d}{dx}\operatorname{sech} x = -\tanh x \operatorname{sech} x$
$\frac{d}{dx}\operatorname{cosech} x = -\coth x \operatorname{cosech} x \ (x \neq 0)$
$\frac{d}{dx}\ \sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$
$\frac{d}{dx}\ \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}} \ (1 < x)$
$\frac{d}{dx}\ \tanh^{-1} x = \frac{1}{1-x^2} \ (|x| < 1)$
$\frac{d}{dx}\ \coth^{-1} x = \frac{1}{1-x^2} \ (1 < |x|)$
$\frac{d}{dx}\ \operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}} \ (0 < x < 1)$
$\frac{d}{dx}\ \operatorname{cosech}^{-1} x = -\frac{1}{|x|\sqrt{1+x^2}} \ (x \neq 0)$