WESTERN SYDNEY UNIVERSITY

Effective use of Zoom for Teaching Mathematics

Tips for delivering mathematics lectures and tutorials via Zoom

MATHEMATICS EDUCATION SUPPORT HUB (MESH)

Contents

Teaching aims

- Students master content
- Students happy and engaged members of class

In all our teaching, we aim for students to:

- master course content
- be happy, engaged members of the class.

In all classes, we want all students to feel safe and comfortable, and to be actively engaged in their learning process.

In order to be able to learn students need to:

- feel safe and comfortable in the classroom
- be engaged in their learning
- know how to get help.

Online teaching challenges

- Can't observe students
- Very limited board space
- Discussions can be more difficult
- Everything takes longer

There are constraints in real-time online lessons due to the online technologies.

Here are some of them:

- variable internet connection quality
- difficult to obtain feedback from students
- difficult to provide feedback to students
- difficult to observe student work
- very limited board space
- handwriting is difficult
- conversations and discussions are difficult with more than about 4 people
- limited to oral/aural and visual communication
- everything takes more time online.

There are also differences in the number of senses that can be used. In a face-to-face class, lessons can involve all the senses – sight, sound, touch, kinaesthetic, and even taste and smell! Online, you are limited to visual and oral/aural communication.

Lessons that were effective in face-to-face classrooms may need to be adjusted in order to run effectively online.

Effective speaking

- Speak Very Clearly
- Write down the important bits

Problems

- Internet quality can be variable. The audio may fade out or become unclear from time to time.
- Students may be in noisy environments – near a main road, a construction site, or on a flight path, that makes it harder for them to hear.
- Students may not be able to see you. This means that they are missing all your visual cues (body language) that they would get in a face-to-face class.

What to do

- Enunciate carefully. Speak clearly and not too quickly.
- Repeat key points in case they were missed the first time.
- Write down the key points.
- Check regularly that your audience can still hear and understand you. If you are regularly interacting with them, this will happen automatically.

Effective visuals

- Pre-prepare visuals to optimise board space
- Write succinctly
- Use highlighting and pointers to direct attention

Optimising board space

Board space is very limited in the online classroom. Running out of room halfway through a proof or a solution and squeezing the rest into a gap on the side is unhelpful.

Here are some options for optimising use of your board space.

- Load lecture notes into Microsoft OneNote or similar to obtain unlimited amounts of notepaper (for worked examples and explanations) beside the lecture notes.
- Pack lecture slides less densely to allow more room for brief annotations.
- Don't write over printed slide notes. No one can read it. Don't scrawl notes in any small gap between the notes. They don't make sense when viewed afterwards. If you are out of room on the slide, you need to go somewhere else to write.
- Include blank slides in your presentation where you can write examples and annotations.
- Pre-prepare visuals for tutorials so that a single question displays at a time, with plenty of room to display or work through the solution. Don't display the tutorial worksheet with all the questions listed: it leaves you with no room to work.
- If you are going to be writing during the class, practise what you are going to write to work out how to best set it out on the online board.
- Pre-prepare everything so you don't need to write during the class.

Handwriting is difficult

Here are some suggestions.

- Look for ways to minimise the amount of writing you need to do.
- Prepare succinct solutions and give the extra explanation orally.
- Pre-prepare written work (in word, Latex, OneNote) and then display it during the class rather than writing it up.
- Factor the extra time needed for writing into your lesson.

Visuals to support learning

- Use highlighting and pointers to direct attention.
- If using complex visuals, take the time to explain each bit of them. Students cannot concentrate simultaneously on figuring out a complex slide and listening to you.
- Keep visuals and text labels close together on the screen. If they are on different parts of the screen, this divides attention.
- Use visuals that minimise cognitive load
    - use audio to explain visuals rather than written text
    - less is more (e.g., do not add redundant text that is a repetition of the audio), omit extraneous visual 'noise'
- Use graphics that build mental models – get students to explain and interpret the graphics.
- Select visuals that communicate the relevance of the lesson content.

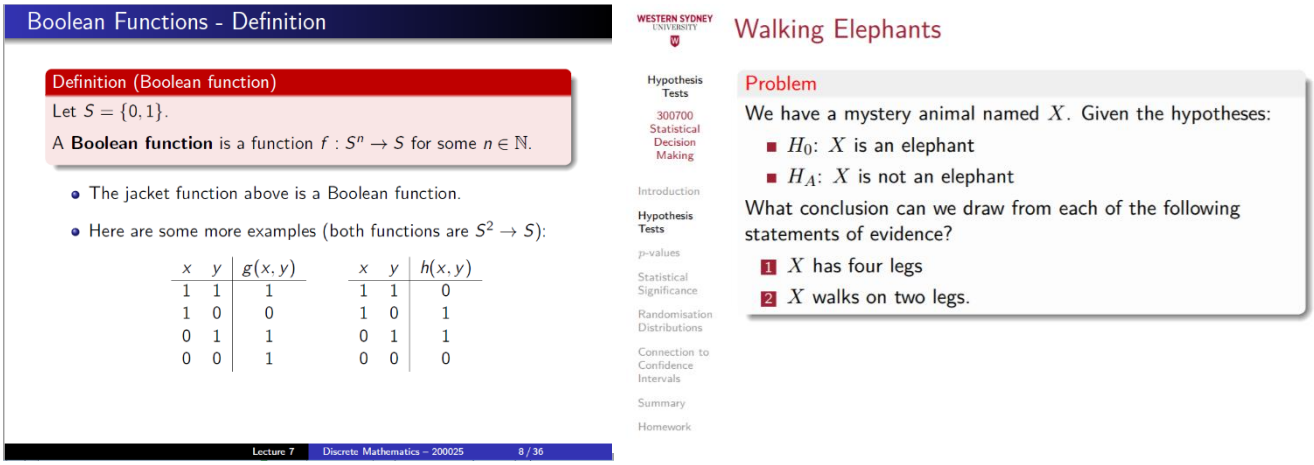
Design visuals that work in the virtual classroom interface

- Visual real estate is very small and may only occupy part of the learner's screen.
- Plan visuals that are simple.
- For complex graphics, use a series of slides that builds or use an overlay technique. This change of scenery will have additional advantage of holding learner attention.
- Text on whiteboard – no smaller than 16 points.
- Emphasise explanatory graphics.
- Limit amount of text to key points.

Lecture slides

- Need a clear structure
- Use colour consistently
- Uncover slides bit-by-bit
- Don't crowd slides

- Lecture slides need to have a very clear structure. Make it easy for students to see the structure. Distinguish visually between the levels of the structure. Examples of structure might include levels such as overview, the key ideas and details or theory, propose, worked examples and practice questions.
- It should be easy to identify theorems, definitions and worked examples.
- You can achieve better student outcomes by using incomplete lecture notes – notes with blanks in them where students have to fill in key words and concepts – this forces students to more actively engage with the lecture, and draws their attention to the most important bits.
- If using slides, displaying the whole slide and talking through it is less effective than bringing the content up one line at a time. *When you look at mathematics on a screen, it's difficult to know where to focus – you can't scan the text – and it distracts you from listening, so it is very easy to lose focus. It is far easier to stay focused when the text appears synchronously with the spoken word.*
- Slides should not be too dense. There should be enough room for brief annotations.
- Use highlighting (coloured text, bold, underline, italics) to emphasise key words and concepts, not words on which you would place spoken emphasis. *Highlighted content provides a summary of the slide.*
- Slide titles should provide a summary of the slide within the context of the lecture e.g. "Differentiation – The Chain Rule – Another Example"
- Readers should understand the context of the slide from the slide itself. This can be achieved through various means, such as:
    - slide headers and footers
    - colour-coding (e.g. definition slides red, example slides green, exercise slides blue)
    - page numbers
    - table of contents bar (although this can clutter the slide).



Boolean Functions - Definition

Definition (Boolean function)

Let $S = \{0, 1\}$.

A **Boolean function** is a function $f : S^n \to S$ for some $n \in \mathbb{N}$.

- The jacket function above is a Boolean function.
- Here are some more examples (both functions are $S^2 \to S$):

x	y	$g(x,y)$
1	1	1
1	0	0
0	1	1
0	0	1

x	y	$h(x,y)$
1	1	0
1	0	1
0	1	1
0	0	0

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Hypothesis Tests

300700 Statistical Decision Making

Introduction
Hypothesis Tests
p-values
Statistical Significance
Randomisation Distributions
Connection to Confidence Intervals
Summary
Homework

Walking Elephants

Problem

We have a mystery animal named X. Given the hypotheses:

- H_0: X is an elephant
- H_A: X is not an elephant

What conclusion can we draw from each of the following statements of evidence?

1. X has four legs
2. X walks on two legs.

A couple of different lecture slides

Involving students

- Get students actively involved as much as possible

It is very easy for students to disengage when watching a lesson online, particularly if they are being passive. However, if students are actively involved then they are paying attention.

Some ways to get students actively involved are:

- initiate frequent interactions with students
- have students present content
- use breakout rooms for small group activities.

Effective interactions

- Interact often
- Use a variety of interactions

Initiate frequent interactions

- Frequent varied interactions are very important because.
  - They keep students focussed and less likely to multi task.
  - They break up the cognitive loading.
  - They let you know that your Internet connection is still good.
- Regularly ask students for responses (via poll, typed response, whiteboard annotation, audio). In tutes, ask for responses every 1-3 minutes. In lectures, asking every 10-20 minutes is more appropriate. Wait until you get a response before continuing.
- Activate prior knowledge with interactions (e.g. interactive reviews of previous lessons; ask each student to summarise a problem they have faced)
- Manage cognitive load by staging interactions to provide cognitive breaks for the students.
- Use breakout rooms for small group activities.

Online methods for interacting

Here are some options.

- Verbally
  - Best for longer open-ended questions (e.g. explain reasons for your answers)
  - Audio participation increases social presence (so you should use some)
- Zoom chat
  - Good for short questions and responses
  - Ask learners to answer type answer into chat. If you wish, you can ask them not to hit send until you tell them to. Alternatively, all type in answer but only ask one person to press send.
- Zoom private chat
  - Private chat (with guidelines) for pairs to formulate an answer.
  - Can be used to communicate privately with you.
- Zoom icons – thumbs up, thumbs down, raise hand – to indicate requests and responses

    - Pose a question and ask for 'hands up' or include calling on a particular participant
- Share screens – students share screens of their work
- Polling – via a Zoom Poll or other online polling program
    - Multiple choice questions with anonymous responses. The entire class can see the results. Very useful for deciding which questions or topics to work through in an instructor-led tutorial.
- Collaborative work on a whiteboard
    - Icebreaker tip: Participants mark a location on a map and practice using annotation tools
    - Ask learners to highlight or annotate the shared whiteboard
    - Students can collaboratively annotate shared whiteboards, in breakout rooms or in main room
    - At end of class ask students for anonymous feedback onto whiteboard in two columns: positive and negative.
    - Whiteboards can be saved.
- Breakout rooms
    - 2 to 5 participants in each (groups spokesperson to summarise to whole class)
    - copy and paste white board responses to main room)
- Application sharing

Online discussions are difficult with more than about four people

Here are some options.

- Use breakout rooms with around four people in each.
- Appoint a discussion leader (this will be you in an instructor-led session) to manage the discussion.
- Use Zoom "raise hand" icon to select people to speak.
- Use the Zoom Chat function for the discussion.
- Use the Zoom Chat function for people to indicate their input, then invite individuals (by name) to speak via microphone.

Obtaining student responses

- Persistence pays off. Keep asking.
- After you ask, wait for an answer.
- Use effective, varied questions.

Soliciting feedback

Obtaining feedback is extra hard when you cannot see your students.

- Persistence does pay off. Keep actively soliciting feedback regularly throughout the class.
- You can:
  - ask for answers to a question
  - ask students if they understand
  - ask if students have any questions.
- Positively reinforce students when they do interact.
- Use an anonymous response method, e.g. whiteboard annotation.
- Specific questions/directions work better than more general questions/directions.
- More effective questions/instructions:
  - What theorem do we need to use here? Write its name in the chat if you know, otherwise write “don’t know”
  - Do this calculation on your calculator and write your answer in the chat.
  - I want to see if I have explained this clearly enough. If you this makes sense to you, press the thumbs up button. If you don’t understand, press the thumbs down button.
  - Read all of the questions on the sheet. Choose one to work on now. Write your choice into the chat.
  - I have set up a poll for you. Please select the three questions that you most want to go over in this class.
- Less effective questions/instructions:
  - Look at Q3. What’s the answer?
  - How do you do Q5?

    - Have a go at the problems.
- Asking “What questions do you have?” can work better than “Do you have any questions?” because it assumes that the students should have questions.
- Make sure students know that they can message you with questions privately in chat, and can annotate screens anonymously. This makes some students feel more comfortable.
- If you have a really silent tutorial class where you struggle to get any student response, try starting out by doing one question to give the students time to settle in before you ask them to interact. Begin by asking them simple questions (e.g. for the numerical answer to the problem you have just done for them, or for a theorem definition that they need to use in the problem).
- It is fine to ask students by name for an answer, but be prepared with a plan B if they don’t respond. Be careful not to always end up asking the same few students because you know they will respond.

Effective questions

- Use closed-ended questions to get specific bits of information
- Use open-ended questions to expand on answers
- Use varied questions.

Closed-ended questions

- Used to elicit specific information.
- Useful for directing students.
- Useful for fast interactions.
- Useful to get students interacting and starting discussion.
- Useful for checking understanding.

Open-ended questions

- Used to expand on answers to closed-ended questions.
- Useful for allowing students to explain their understanding.
- Useful for stimulating students to higher-order reasoning and understanding.

Direct questions to specific students or to the class generally. In a large online group, most questions should be fast, close-ended questions directed to the class in general, with a means for them all to respond (e.g. via chat, or icon selection). General questions include all students, whereas if you direct all questions to specific students, the rest of the class may tune out. There is also the risk that the selected student may not answer, and you need to know what you are going to do then. You don't want to end up directing all your questions to the three or four students you know will respond as this effectively excludes the rest of the students. On the other hand, if a student is being very quiet, a question directed to them may be very appropriate.

Examples of closed-ended questions:

When visiting a breakout group:

- "*What answer did you get for question 1?*" (This assumes that they have been working on the material and have completed Q1. If they haven't they will need to justify why to you. If they have, then you get to discuss the maths with them. This is a better question than

"How are you going?" or "What question are you working on?", or "do you need any help?"

In a teacher-led session:

- *"Will the answer be more or less than 6.3? Write your answer in the chat."*
- *"What answer did you get for Q2? Write your answer in the chat."*
- *"Annotate the whiteboard and mark your answer on the number line."*
- *"There are a number of methods you could use to get to the answer. What method did you use to get to the answer? Write up your method on the shared whiteboard."*
- *"What theorem could we apply here? Write your answer in the chat."*
- *"I have set up a poll for you. Please choose the three questions that you most want me to go through. When you've all voted, we will go through the questions in the order you selected."*

Examples of open-ended questions

When visiting a breakout group:

- (Having seen some group work on the shared screen): *"<Student Name>, can you explain your solution to this question to me?"* This is more effective than "How did you get from here to here?" Students may need to explain the working from the beginning in order to be able to explain a step halfway through the solution. This way you can see exactly what they do understand, and you can ask more questions of them, and of the group, as they go through the explanation.

In a teacher-led session:

- *"Why do I need to put a minus sign here?"*
- *"I see that we have a few different answers for this question. Could someone explain how they got 5.*ln*(π+3)?"* (And then ask someone to explain the other answer. The students should hopefully get involved and decide which is the correct answer themselves while you sit back and watch.)

Vary your questions

- Mix up your questions. Ask yes/no questions, give-me-the-correct-answer questions, short answer questions, opinion questions, how-do-I questions, why questions, what-would-happen-if questions and whatever else seems appropriate.

- Vary the questions, but don't use too many response modes. Don't do one by poll, then one by chat, then one by icon, etc. Pick a response mode and use mostly it (e.g. chat or annotation and/or voice).

Effective breakout rooms

- Students can use the technology
- Small, diverse groups
- Structured activities
- Clear directions
- Clear, essential roles for each and every person
- Every group has a deliverable
- Tight time constraints

- Make sure all students can use the technology before trying to do maths in the breakout room sessions.
- Provide very clear – written and oral - directions for what is to happen in the breakout room.
- Keep groups small and diverse (3 to 5)
- Structure every activity. Set it up so that each group member has an essential role (e.g. Go into your breakout rooms. Discuss, and then vote on which of the three solutions is best. Bring your choice back to the main room)
- Give every person in the group an indispensable role in the group (e.g., each member of the group is to state their choice and reasons, then the group is to vote on its choice, or; each group of four requires a facilitator, two scribes working in different colours, and a presenter).
- Keep activity focussed and brief (e.g. 3-4 min for a discussion).
- Provide a template for each BR whiteboard.
- Each group should copy BR whiteboard to bring back to main room.
- There must be a deliverable outcome from every breakout group (e.g. the group's vote, the group's whiteboard, the group's set of working and solutions).
- Assign a facilitator/spokesperson for each group, or make the group choose one as its first task.

Time management

Everything takes more time online ...

... so plan for it!

You need to factor this in when preparing the lesson. Rushing through a lesson to cover all the content does not make a good lesson. Choose what you will sacrifice; don't let running out of time near the end of the lesson decide for you.

Classroom management

- Explicit expectations
- Create routines
- Use micro-scripts
- Always be calm, friendly and enthusiastic

Our observations have been that good classroom management practices are even more important in the online classroom than the face-to-face.

Getting off to a good start with your students. Make expectations explicit, explicitly teach class routines, and prepare micro-scripts to use on students when you need to (e.g. a student turns up half an hour late but wants to be marked as present for the tute). Be respectful and enthusiastic, this can make the difference between a successful class and a not-successful class.

Be explicit about expectations for the class

- Select just a few essential expectations. You can reasonably have up to three expectations.
- Complying with these expectations is not negotiable in the class. For example, if they are meant to be working in groups on a problem set, that is what they should be doing. When you visit the groups, ask questions that emphasize this expectation, e.g., “which questions have you completed so far? … Show me your working for them.” (This forces them to discuss what they have actually done and leads into discussing the work) instead of “hello, how are you going?” (To which they will answer, “Fine.” End of conversation.)

Create routines

- Explicitly teach the routines (e.g. if you are running a whiteboard tutorial and you want students to work in groups of 3 and write their names up on the board and each use a different colour pen, then for the first couple of weeks focus your attention on this above all else. Only help with the teaching and learning when they are complying with this.
- Be completely explicit about what you expect. Don’t leave students to guess.
- Once you have a routine that allows all the logistics to run smoothly then you end up with more time to focus on the course content.

Use micro-scripts

- Know in advance, what you are going to say to students in various situations. For example, if a student shows up 15 minutes late, what are you going to say to them? Have a short script prepared to use. This ensures you are prepared and consistent.

Pay attention to how you come across

- Enthusiasm is contagious. So is disengagement. It is important that you display high levels of enthusiasm for your class (even if it is just an act!).
- Stay calm. Teaching spaces where emotional responses from the teacher are the norm can be difficult and frightening places to learn.
- Be emotional when praising students. Be emotionless and mechanical when censuring them.
- If possible (e.g. in tutes), greet each student by name.
- Make your learners feel important, welcomed and appreciated right from the start and you have every chance of a successful lesson.
- Human behaviour is reciprocating. Whatever you want from your students you need to do to them (smile, listen, show respect, etc.).

Effective online pedagogy

- Arouse curiosity
- Activate prior learning
- Manage the cognitive load
- Guide student learning
- Active learning

Good pedagogical techniques are very important online where you have less control over your students and far less feedback from them since you can't see them.

Start with lead-in questions

- Start each lesson with one or two topic lead-in questions.
- These stimulate interest and activate previous knowledge.
- These can be short discussions (show a visual or pose problem), or you can ask students to analyse in small groups then come back to discuss in main room
- You can use topical questions, or rhetorical questions, or obvious questions, or completely unexpected questions that seem unrelated to the topic.
- Using topical questions takes a little longer as they are open-ended.
- Rhetorical questions, or obvious questions (e.g. "who drinks water?"), or unexpected questions (e.g. "I like to shower. Do you?") Do not require students to actually respond. However, students will answer mentally, which means that they are engaging with you. And they won't know where you are going with this opening, so they will listen in order to find out. Then you have a minute or two to show them how this opening question is relevant to (indeed, at the heart of) today's lesson.
- Whatever your question, it needs to (a) arouse curiosity, and (b) activate prior learning.

Manage the cognitive load

- Explicitly teach participants to use response tools (during first class).
- Provide clear and visible (written) directions along with questions.
- Offer visual memory supports (e.g. stick the procedure or formula up on the screen.)
- Avoid extraneous interactions.

- Gradually progress from demonstrated examples to student practice. Demonstrate, then do several examples where learners complete it (faded worked examples) gradually learners do more and more of the steps and you do fewer.

Activate prior knowledge and consolidate learning

- Teach to integrate new knowledge with existing knowledge.
- Use lesson introductions to bring out relevant information from long-term memory.
- Give brief lesson reviews at the end of each lesson.
- Visuals can be helpful in this.

Promote knowledge transfer

- Embed appropriate retrieval hooks or cues in exercises.
- Use a variety of experiences that let learners try out content in diverse settings.

Guided learning

- Most students are not good judges of what they do and do not know and what they should study. They need to be told.
- Pre-tests (which can be quite informal) can show students what they do and don't know.
- Recommend helpful websites.
- Assess learner progress.
- Use participant responses to 'read' audience progress in the absence of body language cues.

Do some activities that let students learn for themselves

Sometimes let students experiment, explore and practise with examples to figure out things for themselves, rather than telling them directly.

- Needs carefully selected examples and or non-examples that illustrate the main concept.
- Ask students to study examples and derive the definition rules or guidelines.
- Such lessons take longer to prepare, save them for the most important topics.
- Wrap up the session and embed student responses into the formal definition.

Increase your ratio of closed to open ended questions

- Start with a closed question - use polling - and follow with open discussion of reasons for choices.
- Call on students for responses to open-ended questions and elaborations on closed-ended responses.

Rely on inclusive rather than individual response options

- Favour responses options that are inclusive rather than individual (e.g. polling and chat over audio) or for small classes – whiteboard.
- The occasional use of audio (i.e. you speaking) is recommended – this maintains social presence and keeps participants alert as they may be called on any time to contribute.
- If a student asks a question, ALWAYS paraphrase and repeat the question. Ask or say something to include the other listeners (e.g. “who else had this question?” or “many people ask this question – it’s an important one to deal with …”). This keeps the Q&A relevant to the whole class. If the question is not relevant to the whole class, tell the student that you will talk to them after the class.

Extend the virtual classroom with assignments

- Assign review and analysis of more complex examples as an offline activity… (include a worksheet for students to complete and email to the instructor, or show to the instructor in the next class).
- Announce a four-hour session where the second two hours is for independent or group work in, or external to, the virtual classroom session.
- Provide a place for asynchronous discussion.
- Assign a leader for each group.

Course design

- Model course skills
- Model mathematical reasoning
- Model mathematical communication

- In mathematics subjects, we want students to master the course content. We also want them to develop their mathematical reasoning and their mathematical communication skills.
- An essential component of developing these skills is seeing them modelled.
- Part of the course design is deciding where this is going to occur.
    - Will tutors be giving instructor-led classes where they model this?
    - If tutorials are comprised of student group work, will lecturers be modelling through worked examples during lectures?
    - Alternatively, will there be extremely high standard worked solutions that model the finished solution, and annotate the reasoning required to get there?
- Make these choices explicit to teaching staff and students.

Lecture structure

- Break every 20 min
- Chain knowledge between sections
- Manage the cognitive load

Structure

Having a good lecture structure becomes more important in an online setting in which students are by themselves without their peers to support and motivate them, and where you cannot easily gauge how students are going because you cannot see them.

Breaks

- Build in regular breaks. Aim for a break every 20 minutes. This is the optimal length of time for people to concentrate on listening to a speaker.
- Choose appropriate places for breaks – e.g. at the end of sections, or when the cognitive load has been very high.
- The breaks do not need to be coffee breaks. Thirty seconds to a minute is sufficient. Ask the students a question that gets them moving (type in an answer, or select an icon). Make an announcement. Tell a mathematician's biography. Ask the students to do a quick problem.
- These breaks help to manage cognitive load.
- Minimise the number of breaks in which students have time to walk away from the computer. Every time that a student walks away from the computer, there is a temptation not to come back. Unless you have a three-hour lecture in which you need walk-around breaks, break, but keep the students engaged online.

Best practice structure

A best practice umbrella structure for lectures triggers prior knowledge, provides many hooks for the day's knowledge to stick to, and chains the knowledge together so that students more easily recall it.

1. Big picture
2. Where today's material fits into the big picture
3. Review of last lecture
4. How today's material follows on
5. Today's material
6. Review of today's material
7. How today's material fits into the big picture
8. Preview of tomorrow's lecture

- The big picture provides students with a framework within which to put the new learning. This makes it easier for them to learn the content, and to link it to other learning that they already have.
- The review and the preview chain the knowledge, which strengthens links between the new content, which makes it easier to access, and helps learners develop deeper understanding. It also contextualizes the new content.
- The review also consolidates previously taught material.
- The preview and review don't need to take more than a minute or two each, unless there is a good reason to extend them (e.g. the first lecture on a new topic). An outline slide or a diagram can be sufficient to illustrate the big picture and where today's content fits in.
- You can place announcements and any other content wherever you like.
- Short anecdotes about the history of the mathematics or a very brief biography of a relevant mathematician can provide a human dimension to your lectures that some students respond well to.

Tutorial formats

- Instructor-led sessions or student-led group work — both can be effective
- Students should be as actively engaged as possible for as much of the time as possible

Instructor-led problem solving

- It is important to give students some control over the tutorial – for example, by asking them to nominate the problems to be done during the tutorial, and by getting them to provide their ideas, solutions and methodologies.
- It is important to continually solicit feedback from students during the tutorial in a patient and encouraging manner.
- One idea that can work to get over the long, awkward silences at the start of the lesson when no students respond to anything is for you to start working through a problem. As you go, you can start asking simple questions for them to answer in the chat, and gradually ask for more and more input, until students are settled and participating.
- You can ask students by name for their response, but you need a plan B in case they don't answer. If you do this, be careful not to distress students by embarrassing them publicly.
- Giving students "a few minutes" to do a question – silently, online – is not effective. "A few minutes" ends up being less than one minute because it feels like a long time when you are waiting silently, but it is not long enough for students to do the problem. It is not a good use of the communal time. When you do this in a classroom, there is some collaboration – students see what other students are doing – but online everyone is on their own and there is no benefit to spending class time on individual work that you cannot see, and cannot provide feedback on.
- It is important to model what you want the students to do. It is not enough to talk through the reasoning used to solve a problem. You need to:
  a) Talk through where in the lecture notes/textbook to refer to / where to find information on the theory you need to do the problem – it's not your job to go through all the theory, but students do need to know how to look it up for themselves.
  b) Discuss how to approach the problem.
  c) Write up the solution in a way that would get full marks.

- It is a bad idea to cut corners when writing up problems in tutorials. It is better to make explanations more succinct and write out a full, beautiful solution than to give lengthy amazing explanations with an incomplete written solution or just a few calculations scribbled down. Students need the whole mathematical process – including mathematical communication – modelled for them.
- Ask the students straightforward questions ("What theorem do we use here?" "How do we simplify this?"), but also ask them higher-order questions ("How does this work on Boolean Algebra relate to our previous section of work on Sets?" "Could you have solved this problem without using Calculus?" "What are the implications of this result?"). Higher order questioning leads to deeper understanding.

Student group-work tutorials

A number of different approaches are in use:

1. Students solve new problems in breakout room groups
2. Students have pre-solved problems reviewed in breakout room groups
3. Students have pre-solved a set of problems. They then solve a similar set of problems in breakout room groups.

Method 2 requires students to be put into pre-allocated rooms. Methods 1 and 3 can use either pre-allocated or randomly allocated rooms.

Observations were that presenting students with new problems to solve was more effective than having them review pre-solved problems.

Hence, method 2 was not as effective as the other two methods. In method 2, students did not make good use of the group time. Mostly they chatted or waited silently until the tutor came to assess their work. They did not discuss their work amongst themselves. After tutor assessment, they either went back to chatting/waiting silently, or left the class. There were also indications that the whole group often did not work through the pre-solved problems.

The main factor in how effective the other group-work tutorials were was dependent on the effectiveness of the tutor's methodology. Good classroom techniques – clear expectations, explicit routine, effective questioning, and approachable demeanour – appeared to be by far the most important factor.

It seemed unimportant whether students were pre-allocated or randomly allocated to their groups.

Presenting worked solutions

- Pre-prepare solutions
- Keep solutions succinct
- Provide complete solutions

- Hand writing solutions onto a small screen is challenging. Here are some effective approaches.
    - Pre-prepare the worked solution. Have it succinctly written out in front of you. Then take the time to write it up neatly and carefully in a way that makes it fit nicely onto the board.
    - Share a whiteboard and collaboratively work up the solution with the students.
    - Write up the worked solution in MSWord, then cover the text with many white rectangles, and uncover the rectangles one by one as you work through the solution.
    - Write up the worked solution in LaTex, using the Beamer uncover command, then uncover the solution a little at a time as you work through it.
    - Write up the solution in One Note, or handwrite and scan the solution, then use a "gradually uncover" approach to display the solution in the class. You can achieve this by covering the solution with a white rectangle that you gradually move, or remove.
- Keep written solutions succinct. Don't write more words than you need to. Save the extra words for your oral explanation.
- Ensure solutions are complete. Model solutions that will get students full marks. Do not cut corners in written solutions, or omit bits.

Guiding students to deeper understanding

- Ask questions that link current learning to prior learning
- Ask students for other ways to do a problem
- Ask students for other applications for a technique

The way to guide students to deeper understanding is to ask them questions that make connections between different parts of their knowledge.