Logic: AND, OR, NOT

Logic is central to controlling the order in which instructions are executed in computer programs. In this module you will learn to use the logical operators AND, OR and NOT, which allow programmers to specify conditions which determine the flow of control.

True and false

Computer programs contain statements or conditions that can be true or false, so we begin with simple statements that are either true or false (but not both).

Logic is at the heart of computers; it is needed to understand how single bits are manipulated in the CPU. The most basic of operations are logic operations. Logic is needed for programming to ensure that the correct statements are executed, and executed in the correct order. Logic deals with statements or conditions that are either true or false.

Here are some simple statements together with their truth values.

- $1 + 1 = 2$. This is true.
- $45 < 50$. This is true.
- $1 + 1 = 4$. This is false.
- If our friend Jane is 20 years old then the statement "Jane is at least 18 years old" is true.

We sometimes write T for true and F for false.

Practice questions for True and False

The answers are at the end of the document. Attempt each question before looking at the answers.

Find the truth values of

1. $2 + 2 = 4$.
2. The capital city of Australia is Perth.
3. $42 > 24$.

True and false, more examples

This page contains more examples of logical statements and their truth values. Some mathematical notation that is used in this module is also revised.

Some of the examples in this module use the symbols $>$, $\geq$, $<$ and$\leq$. Each can be replaced by words, but they are used because they are short and easy to write.

$>$ means "is greater than"
$\geq$ means "is greater than or equal to"
$<$ means "is less than"
$\leq$ means "is less than or equal to".

Here some examples of statements and their truth values.

- $1 + 2 = 3$.
  This is correct, it is true. Another way of saying this is that the truth value of $1 + 2 = 3$ is true.
- $1 + 2 = 7$.
  This is not correct. It is false.

- The distance from Sydney to Melbourne is 100km.
  This has truth value false. (It is more than 700km from Sydney to Melbourne.)
- $22 \geq 33$ (read 22 is greater than or equal to 33).
  This is false.
- $0 < 6$.
  This is true.

More practice questions for True and False

Find the truth value of each statement (that is, decide whether each is true or false).

1. $5 - 3 = 0$.
2. The sky is always red.
3. $23 \leq 100$.

Answers: F. F. T.

The logical operator AND, &&

Simple conditions or expressions can be combined using the operators AND, OR and NOT to create more complicated and useful expressions. This page introduces the operator AND.

In Java and some other languages the logical operator AND is written &&.

Imagine that you are at an airport and want to board a flight. You might be told that the condition for being allowed to board the flight is that

you have your passport AND you have a ticket.

This tells you that

- You will be able to board if you have both those items.
- If you don't have your passport but do have a ticket you will not be allowed to board.
- If you don't have a ticket but do have a passport you will not be able to board.
- If you have neither you will not be able to board.

We summarise this example in a table.

You have your passport	You have a ticket	You may board
True	True	True
True	False	False
False	True	False
False	False	False

We summarise this in a *truth table*. The truth table gives the truth value of a AND b, that is, a && b, for all possible truth values of the pair a, b.

a	b	a && b
T	T	T
T	F	F
F	T	F
F	F	F

The truth table defines the use of && (AND).

In some places, for example, some maths books, you might see the symbol $\wedge$ used for the logical operator AND.

Examples

- Find the truth value of $24 > 50$ && $44 - 44 = 0$.
  Answer: Firstly, $24 > 50$ is F, $44 - 44 = 0$ is T. We have $\underbrace{24 > 50}_{F}$ && $\underbrace{44 - 44 = 0}_{T}$.
  As F && T is F, the answer is F.
- Find the truth value of $50 > 24$ AND $44 - 44 = 0$.
  Answer: Both $50 > 24$ and $44 - 44 = 0$ are T. We have $\underbrace{50 > 24}_{T}$ AND $\underbrace{44 - 44 = 0}_{T}$.
  As T AND T is T, the answer is T.

Practice questions for AND

The answers are at the end of the document. Attempt each question before looking at the answers.

Find the truth values of

4. $1 + 1 = 2$ && $42 > 24$.
5. The capital city of Australia is Perth AND Western Sydney University is in Tasmania.
6. $42 > 24$ AND $24 > 42$.
7. $1 = 2$ && $2 \leq 4$.

AND, more examples

This page contains more examples of the use of AND.

The logical operator AND is written && when writing a program in Java. Here both AND and && are used because you need to know both.

Often T is written for true and F for false.

Example 1. Find the truth value of $6 - 6 = 0$ && $1 \leq 2$.
Answer: (This is the same as asking for the truth value of $6 - 6 = 0$ AND $1 \leq 2$.)

Firstly we need to determine the truth values of $6 - 6 = 0$ and of $1 \leq 2$: $6 - 6 = 0$ is T (true) and $1 \leq 2$ is T. We have $\underbrace{6 - 6 = 0}_{T}$ && $\underbrace{1 \leq 2}_{T}$. That is, we have T && T.

The truth table for && shows that T && T is T. The answer is T.

Example 2. Find the truth value of $2 + 2 = 10$ AND $1 \leq 2$.
Answer: (This is the same as asking for the truth value of $2 + 2 = 10$ && $1 \leq 2$.)

$2 + 2 = 10$ is F (false) and $1 \leq 2$ is T (true). So we have $\underbrace{2 + 2 = 10}_{F}$ AND $\underbrace{1 \leq 2}_{T}$, which is F AND T. From the truth table for && we see that F && T, which is the same as F AND T, is F. The answer is F.

Example 3. Find the truth value of $2 + 2 = 10$ AND $2 \times 2 = 22$.
Answer: $2 + 2 = 10$ is F and $2 \times 2 = 22$ is F. So we have $\underbrace{2 + 2 = 10}_{F}$ AND $\underbrace{2 \times 2 = 22}_{F}$, which is F AND F. From the truth table for && we see that F && F is F. Therefore F AND F is F. The answer is F.

Example 4. Find the truth value of $1 \leq 2$ && $2 + 2 = 10$.
Answer: We have $\underbrace{1 \leq 2}_{T}$ && $\underbrace{2 + 2 = 10}_{F}$, which is T && F. From the truth table for && we see that T && F is F. The answer is F.

More AND practice questions

Find the truth value of each statement (that is, decide whether each is true or false).

1. $5 - 3 = 0$ && $-1 < 1$.
2. The sky is always red AND Finnish is the main language of Australia.
3. $23 \leq 100$ AND $0 = 0$.
4. $23 \leq 100$ && $0 + 1 = 0$.

Answers: F && T, which is F; F && F, which is F; T && T, which is T; T && F, which is F.

Third set of AND practice questions

Find the truth value of each statement (that is, decide whether each is true or false).

1. Every computer program is written in FORTRAN AND Sydney is in NSW.
2. $20 + 20 = 40$ AND $2 \times 3 = 0$.
3. $6 < 10$ && $10 < 20$.
4. $10 < 6$ && $20 < 10$.

Answers: F && T, which is F; T && F, which is F; T && T, which is T; F && F, which is F.

The logical operator NOT, !

The truth value of a statement can be changed by using the logical operator NOT. This page introduces NOT.

In Java and some other languages the logical operator NOT is written !.

Negating a statement (putting NOT in front) changes its truth value. The negation can often be formed by inserting the word "not" into the sentence at an appropriate place. For example, the negation of "The sky is blue" is "The sky is *not* blue".

The truth table for ! (NOT) is

a	$!a$
T	F
F	T

In some maths books you might see the symbol $\sim$ or $\neg$ used for the logical operator NOT.

Examples

- Find the truth value of $!(24 \geq 50)$.
  Answer: $!\underbrace{24 \geq 50}_{F}$ is $!F$, which is T.
- Find the truth value of $!!44 - 44 = 0$.
  Answer: $!!\underbrace{44 - 44 = 0}_{T}$ is $!!T$. This is $!F$, which is T.

Practice questions

The answers are at the end of the document. Attempt each question before looking at the answers.

Find the truth values of

8. $!(1 + 1 = 2)$.
9. NOT(The capital city of Australia is Perth).
10. $!(24 > 42)$.
11. $!!(24 > 42)$.

NOT, more examples

This page contains more examples of the use of NOT.

The logical operator NOT is written ! when writing a program in Java. Here both NOT and ! are used because you need to know both.

Often T is written for true and F for false.

Putting NOT before a true statement creates a false statement. Putting NOT before a false statement creates a true statement.

Examples

Find the truth value of each statement

- NOT $3 = 3$.

  Answer: $3 = 3$ is T (true). So NOT $3 = 3$ is F (false).

- $!3 = 3$.

  Answer: $!3 = 3$ could be written as $!(3 = 3)$. It is the same as NOT $3 = 3$. Therefore it is F.

- \$1,000 is not less money than \$5.

  Answer: We could write this as NOT (\$1,000 is less money than \$5). We know that (\$1,000 is less money than \$5) is F, so NOT (\$1,000 is less money than \$5) is T.

- $!!5 \leq 6$.

  Answer: $5 \leq 6$ is T. So $!5 \leq 6$ is F. Therefore $!!5 \leq 6$ is T.

More NOT practice questions

Find the truth value of each statement (that is, decide whether each is true or false).

1. $5 - 3 = 0$.
2. $!5 - 3 = 0$.
3. $!!5 - 3 = 0$.
4. $! - 1 < 1$.
5. The sky is NOT always red.
6. $!0 = 0$.
7. $!!23 \leq 100$.

Answers: F; $!F$ is T; $!!F$ is F; $!T$ is F; $!F$ is T; $!T$ is F; $!!T$ is T.

Third set of NOT practice questions

Find the truth value of each statement (that is, decide whether each is true or false).

1. $2 + 2 = 4$.
2. $!2 + 2 = 4$.
3. $!!2 + 2 = 4$.
4. Finnish is NOT the main language of Australia.
5. $!10 < 100$.
6. NOT $10 < 100$.

Answers: T; $!T$ is F; $!!T$ is T; $!F$ is T; $!T$ is F; $!T$ is F.

The logical operator OR, $||$

This page introduces the logical operator OR.

In Java and some other languages the logical operator OR is written $||$.

Jess wants to enrol in a course, but there are conditions for enrolment: a person can enrol if they have passed Maths 1 or if they have passed Stats 1. So Jess can enrol if

Jess has passed Maths 1 OR Jess has passed Stats 1.

This tells you that

- Jess has passed both Maths 1 and Stats 1 means that Jess can enrol.
- Jess passed Maths 1 but not Stats 1 means that Jess can enrol.
- Jess did not pass Maths 1 but did pass Stats 1 means that Jess can enrol.
- Jess failed both means that Jess can't enrol.

We summarise this example in a table.

Passed Maths 1	Passed Stats 1	Allowed to enrol
True	True	True
True	False	True
False	True	True
False	False	False

In general, $||$ (OR) is used as shown in this truth table

a	b	$a \,||\, b$
T	T	T
T	F	T
F	T	T
F	F	F

In some places, for example, some maths books, you might see the symbol $\vee$ used for the logical operator OR.

Example

Find the truth value of $24 > 50 \,||\, 44 - 44 = 0$.

Answer: $\underbrace{24 > 50}_{F} \,||\, \underbrace{44 - 44 = 0}_{T}$ is $F \,||\, T$, which is T. The answer is T.

Practice questions

The answers are at the end of the document. Attempt each question before looking at the answers.

Find the truth values of

12. $1 + 1 = 2 \,||\, 42 > 24$.
13. The capital city of Australia is Perth OR Western Sydney University is in Tasmania.
14. $42 > 24$ OR $24 > 42$.
15. $1 = 2 \,||\, 2 \leq 4$.

OR, more examples

This page contains more examples of the use of OR.

The logical operator OR is written || when writing a program in Java. Here both OR and || are used because you need to know both.

Often T is written for true and F for false.

Example 1. Find the truth value of $6 - 6 = 0 \,||\, 1 \leq 2$.
Answer: (This is the same as asking for the truth value of $6 - 6 = 0$ OR $1 \leq 2$.)

Firstly we need to determine the truth values of $6-6=0$ and of $1 \leq 2$: $6-6=0$ is T (true) and $1 \leq 2$ is T. We have $\underbrace{6-6=0}_{T} \,||\, \underbrace{1 \leq 2}_{T}$. That is, we have $T \,||\, T$.

The truth table for || shows that $T \,||\, T$ is T. The answer is T.

Example 2. Find the truth value of $2+2=10$ OR $1 \leq 2$.
Answer: (This is the same as asking for the truth value of $2+2=10 \,||\, 1 \leq 2$.)

$2+2=10$ is F (false) and $1 \leq 2$ is T (true). So we have $\underbrace{2+2=10}_{F}$ OR $\underbrace{1 \leq 2}_{T}$, which is F OR T. From the truth table for || we see that $F \,||\, T$, which is the same as F OR T, is T. The answer is T.

Example 3. Find the truth value of $2+2=10$ OR $2 \times 2 = 22$.
Answer: $2+2=10$ is F and $2 \times 2 = 22$ is F. So we have $\underbrace{2+2=10}_{F}$ OR $\underbrace{2 \times 2 = 22}_{F}$, which is F OR F. From the truth table for || we see that $F \,||\, F$ is F. Therefore F OR F is F. The answer is F.

Example 4. Find the truth value of $1 \leq 2 \,||\, 2+2=10$.
Answer: We have $\underbrace{1 \leq 2}_{T} \,||\, \underbrace{2+2=10}_{F}$, which is $T \,||\, F$. From the truth table for || we see that $T \,||\, F$ is T. The answer is T.

More OR practice questions

Find the truth value of each statement (that is, decide whether each is true or false).

1. $5 - 3 = 0 \,||\, -1 < 1$.
2. The sky is always red OR Finnish is the main language of Australia.
3. $23 \leq 100$ OR $0 = 0$.
4. $23 \leq 100 \,||\, 0 + 1 = 0$.

Answers: T. F. T. T.

Third set of OR practice questions

Find the truth value of each statement (that is, decide whether each is true or false).

1. Every computer program is written in FORTRAN OR Sydney is in NSW.
2. $20 + 20 = 40$ OR $2 \times 3 = 0$.
3. $6 < 10 \,||\, 10 < 20$.
4. $10 < 6 \,||\, 20 < 10$.

Answers: $F \,||\, T$, which is T; $T \,||\, F$, which is T; $T \,||\, T$, which is T; $F \,||\, F$, which is F.

Precedence (what happens first)

In arithmetic, multiplication has precedence over addition, meaning that multiplications are done before additions. This, and other rules, mean that people and computers interpret mathematical expressions consistently. Logic also has rules so that logical expressions are interpreted consistently.

Simple conditions can be combined using several of the operators AND, OR and NOT to form more complicated conditions. There are conventions about how these more complicated conditions are interpreted.

In arithmetic brackets can be used to show the order in which calculations are to be done. Anything in brackets is evaluated before anything else. In an expression with $\times$ and $+$, if there are no brackets to specify order, $\times$ is done before $+$ as multiplication has precedence over addition. That is, all multiplications are done before additions. So $2+3\times 4$ is evaluated by doing the multiplication first and then the addition:

$$2+3\times 4 = 2+12 = 14.$$

However $(2+3)\times 4 = 5\times 4 = 20$.

Much the same is true in logic:

$$\boxed{\text{Evaluate expressions in brackets first.}}$$

NOT has precedence over AND and OR, so unless brackets indicate otherwise

$$\boxed{\text{NOT is done before AND, OR.}}$$

For example, when finding the truth value of NOT a AND NOT b, evaluate NOT a, evaluate NOT b; only then consider AND.

If in doubt, use brackets to show how an expression is to be evaluated.

Beware: Some programming languages have extra precedence rules or conventions. It is best to use brackets when mixing AND and OR in an expressions as conventions here do vary.

Examples

We begin with a note on notation. "NOT $4>5$" only makes sense if it is read as "NOT $(4>5)$". If we read it as "(NOT 4) >5" or "(NOT $4>$)5" it makes no sense. So it is acceptable to write "NOT $4>5$" as this is not ambiguous.

- Find the truth value of NOT $4>5$ AND NOT $1=1$.

  Answer: NOT $\underbrace{4>5}_{F}$ AND NOT $\underbrace{1=1}_{T}$ becomes $\underbrace{\text{NOT } F}_{T}$ AND $\underbrace{\text{NOT } T}_{F}$ which is T AND F. This is F.

- Evaluate $!(1+2=5\,||\,!\,1=1)$.

  Answer: We must begin with the part of the expression in brackets. $!(\underbrace{1+2=5}_{F}\,||\,!\,\underbrace{1=1}_{T})$.

  We have $!(F\,||\,!T)$; this is $!(F\,||\,F)$, which is $!F$. This is T.

- Evaluate $11<12\ \&\&\ (1+2=5\,||\,!\,1=1)$. Here brackets have been used to show what we must evaluate first.

  Answer: $11<12\ \&\&\ \underbrace{(1+2=5\,||\,!\,1=1)}_{\text{Evaluate this first}}$.

  The expression in brackets is $\underbrace{1+2=5}_{F}\,||\,\underbrace{!\,1=1}_{!T}$ which is $F\,||\,F$ which is F.

The original expression is $\underbrace{11 < 12}_{T}$ && $\underbrace{(1+2=5 \,||\, !\,1=1)}_{F}$, which is T && F, which is F.

Practice questions

The answers are at the end of the document. Attempt each question before looking at the answers.

16. Find the truth values of

    (a) $2+3=5$
    (b) $5>6$
    (c) $(2+3=5)$ && $(5>6)$
    (d) $(2+3=5) \,||\, (5>6)$
    (e) $(2+3=5)$ AND $(5>6)$
    (f) $(2+3=5)$ OR $(5>6)$
    (g) $!(2+3=5)$.
    (h) $(2+3=5)$ && $!(5>6)$.
    (i) $!(2+3=5)$ && $(5>6)$.
    (j) $(2+3=5$ && $5>6) \,||\, 3+3=6$.
    (k) $!!(2+3=5)$.

17. Based on the information given decide whether each part is true or false.
    Ann is 34 years old, female and her annual income is $100,000.
    Adam is 18 years old, male and his annual income is $10,000.
    Ali is 70 years old, male and his annual income is $30,000.

    (a) Ann's age is < 25
    (b) Adam is male
    (c) !(Adam is male)
    (d) Ali's annual income is $> \$20,000$
    (e) Ann's age is < 25 AND Adam is male.
    (f) Ann's age is < 25 OR Adam is male.
    (g) (Adam is 18 years old) || (Adam's income is not $10,000).
    (h) (Adam is 18 years old) && (Ali is 18 years old).
    (i) (Adam is 18 years old) && (Ali is 70 years old).
    (j) Ali's annual income is $50,000 && Ann is 10 years old.
    (k) Ali's annual income is $50,000 || Ann is 10 years old.
    (l) Ann is not 18 years old && Ann is not female.
    (m) (Ali is at least 70 years old) && $\Big[$Ann's age is $< 34 \,||\, !$(Adam's income is $> \$20,000)\Big]$.

Precedence, more examples

This page contains more examples of expressions using several operators.

Expressions must be evaluated in the right order.

Evaluate expressions in brackets first.

NOT has precedence over AND and OR, so unless brackets indicate otherwise

NOT is done before AND, OR.

Examples

Find the truth values of each expression

- $2 = 2 \,||\, !3 = 3$.

  Answer: Note that $2 = 2$ is T, $3 = 3$ is T and $!3 = 3$ is F.
  There are no brackets, so we evaluate ! before $||$:
  $\underbrace{2 = 2}_{T} \,||\, \underbrace{!3 = 3}_{F}$ is $T \,||\, F$. This is T.

- $2 = 2 \,||\, (!3 = 3)$.

  Answer: We must evaluate the ! before the $||$. This is what was done in the previous example because ! has precedence over $||$. The answer is T.

- $!2 = 2 \,||\, 3 = 3$.

  Answer: ! has precedence over $||$, so $!2 = 2$ is evaluated first: $!2 = 2$ is $!T$ which is F. Therefore $\underbrace{!2 = 2}_{F} \,||\, \underbrace{3 = 3}_{T}$ is $F \,||\, T$ which is T.

- $!(2 = 2 \,||\, 3 = 3)$.

  Answer: We must deal with the expression in brackets first. This is $\underbrace{2 = 2}_{T} \,||\, \underbrace{3 = 3}_{T}$. This is $T \,||\, T$, which is T.

  Therefore $!\underbrace{(2 = 2 \,||\, 3 = 3)}_{\text{T}}$ is $!T$ which is F.

More precedence practice questions

Find the truth value of each statement (that is, decide whether each is true or false).

1. $!5 - 3 = 0 \,||\, -1 < 1$.
2. $!(5 - 3 = 0 \,||\, -1 < 1)$.
3. The sky is always red OR Finnish is not the main language of Australia.
4. !(The sky is always red OR Finnish is not the main language of Australia).
5. $23 \leq 100$ OR $!0 = 0$.
6. $!(23 \leq 100 \,||\, 0 + 1 = 0)$.
7. $0 = 0$ && $!(23 \leq 100 \,||\, 0 + 1 = 0)$.

Answers: $T \,||\, T$ is T.
$!(F \,||\, T)$ is $!T$ which is F.
F OR (NOT Finnish is the main language of Australia) is F OR NOT F, which is F OR T, which is T.
$!(F$ OR $!F)$ is $!(F$ OR $T)$, which is $!T$, which his F.
T OR $!T$ is T OR F, which is T.
$!(T \,||\, F)$ is $!T$, which is F.
T && F is F.

Answers to practice questions

1. $2 + 2 = 4$ is true.
2. The capital city of Australia is Perth. False.
3. $42 > 24$. True.
4. $1 + 1 = 2$ && $42 > 24$. True.
5. The capital city of Australia is Perth AND Western Sydney University is in Tasmania. False.
6. $42 > 24$ AND $24 > 42$. False.
7. $1 = 2$ && $2 \leq 4$. False.
8. $!(1 + 1 = 2)$. False.
9. NOT(The capital city of Australia is Perth). True.
10. $!(24 > 42)$. True.
11. $!!(24 > 42)$. False.
12. $1 + 1 = 2 \,||\, 42 > 24$. True.
13. The capital city of Australia is Perth OR Western Sydney University is in Tasmania. False.
14. $42 > 24$ OR $24 > 42$. True.
15. $1 = 2 \,||\, 2 \leq 4$. True.
16. (a) $2 + 3 = 5$: T.
    (b) $5 > 6$: F.
    (c) $\underbrace{(2 + 3 = 5)}_{T}$ && $\underbrace{(5 > 6)}_{F}$: F.
    (d) $(2 + 3 = 5) \,||\, (5 > 6)$: T.
    (e) $(2 + 3 = 5)$ AND $(5 > 6)$. This is the same as 16c: F.
    (f) $(2 + 3 = 5)$ OR $(5 > 6)$. This is the same as 16d: T.
    (g) $!(2 + 3 = 5)$: F.
    (h) $(2 + 3 = 5)$ && $!(5 > 6)$: T.
    (i) $!(2 + 3 = 5)$ && $(5 > 6)$: F.
    (j) $(2 + 3 = 5$ && $5 > 6) \,||\, 3 + 3 = 6$ is $(T$ && $F) \,||\, T$ which is $F \,||\, T$, which is T.
    (k) $!!(2 + 3 = 5)$ is $!!\,T$, which is T.
17. (a) Ann's age is < 25: F.
    (b) Adam is male: T.
    (c) !(Adam is male): F.
    (d) Ali's annual income is $> \$20,000$: T.
    (e) $\underbrace{\text{Ann's age is } < 25}_{F}$ && $\underbrace{\text{Adam is male}}_{T}$: F && T is F.
    (f) Ann's age is < 25 OR Adam is male: $F \,||\, T$ is T.
    (g) (Adam is 18 years old) || (Adam's income is not \$10,000): $T \,||\, F$ is T.

(h) (Adam is 18 years old) && (Ali is 18 years old): T && F is F.

(i) (Adam is 18 years old) && (Ali is 70 years old): T && T is T.

(j) Ali's annual income is $50,000 && Ann is 10 years old: F && F is F.

(k) Ali's annual income is $50,000 || Ann is 10 years old: $F \,||\, F$ is F.

(l) Ann is not 18 years old && Ann is not female: T && F is F.

(m) $\underbrace{\text{Ali is at least 70 years old}}_{T}$ && $\left[\underbrace{\text{Ann's age is} < 34}_{F} \,||\, !\underbrace{\text{Adam's income is} > \$20,000}_{F}\right]$:
T && $[F \,||\, !F]$, which is T && $[F \,||\, T]$. This is T && T, which is T.