Approximations: rounding and truncation

Contents

There are many times when computations are done with approximations to numbers, sometimes by choice, sometimes because there is no other way. In these pages we look at two ways of approximating numbers: rounding and truncation.

1   Truncation

Truncation is a method of approximating numbers. It is easier than rounding, but does not always give the best approximation to the original number. Truncation is used in computing when division is done with integers and the answer must be an integer.

The number e is
$$e = 2.718281828459045\ldots.$$

Sometimes a number is approximated by just ignoring digits. This is called *truncation*.

Examples

- e truncated to give an integer is 2;

- e truncated to give a number with 1 decimal place is 2.7;

- e truncated to give a number with 2 decimal places is 2.71;

- e truncated to give a number with 3 decimal places is 2.718;

- $\pi = 3.14159\ldots$ truncated to give an integer is 3;

- $-\pi = -3.14159\ldots$ truncated to give an integer is -3;

- $\pi = 3.14159\ldots$ truncated to give a number with 3 decimal places is 3.141.

Practice questions

1. Truncate 9.15 to an integer (whole number).

2. Truncate 9.15 to give a number with 1 decimal place.

3. Truncate 234.876 to an integer.

4. Truncate 234.876 to give a number with 2 decimal places.

5. Truncate -234.876 to give a number with 2 decimal places.

2   More on truncation

Here some more examples of truncation.

- Truncate 556.817 to give a whole number.
  Answer: It is 556. We just ignore the digits after the decimal point.

- Truncate -556.817 to give a whole number.
  Answer: It is -556. We ignore the digits after the decimal point.

- Truncate 13.42 to give a whole number.
  Answer: It is 13 because we ignore the .42.

- Truncate -13.42 to give a whole number.
  Answer: -13.

- Truncate 0.9765625 to give a number with (a) 1 decimal place, (b) 4 decimal places, (c) 6 decimal places.
  Answer: (a) The answer is 0.9. (b) The answer is 0.9765. (c) The answer is 0.976562.

More practice questions for truncation

6. Truncate 1 026.3847 to give an integer.

7. Truncate $-1\,026.3847$ to give a whole number.

8. Truncate 54.78 to 1 decimal place.

9. Truncate -54.78 to 1 decimal place.

10. Truncate 2.55 to (a) give a whole number, (b) 1 decimal place.

3   Rounding

Approximating numbers by rounding is not quite as straightforward as truncation, but the approximations are better.

The number π (pi) is

$$\pi = 3.14159265358979323846264338327950288\ldots.$$

The decimal goes on forever, but in many calculations we only use a few digits. Computers do not always work with exact values, and π is a good example of a number whose exact value is not used when computers or calculators are doing the calculations. Approximations are used for numbers with infinite or very long decimal expansions. Sometimes approximations are used because keeping extra decimal places is not needed.

Writing π to 2 decimal places means we write $\pi = 3.14$ and do not bother with the other digits. We write only 2 digits after the decimal point.

If we want π to 4 decimal places we could write 3.1415, but π is much closer to 3.1416. We *round* π to 4 decimal places. This means that we use 4 digits after the decimal point, but we use the number that is closest to the real value of π. When we *round* a number we choose the number with the correct number of digits which is closest to the original number.
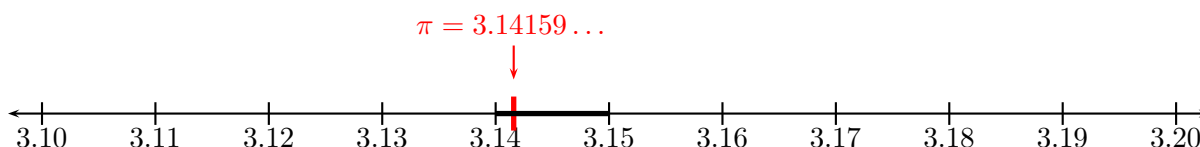
Examples

- π rounded to 0 decimal places is 3 (3.1...is closer to 3 than to 4);

$\pi = 3.14159\ldots$

- π rounded to 1 decimal place is 3.1 (3.14...is closer to 3.1 than to 3.2). To see this, zoom in on the segment from 3 to 4 in the line above:
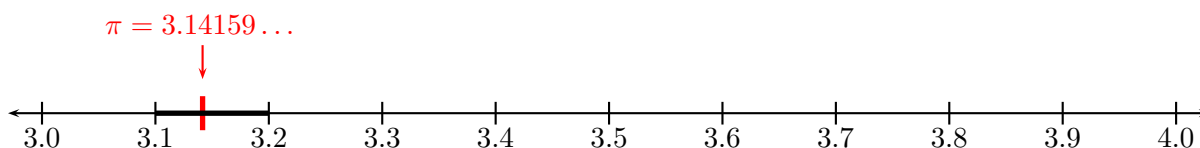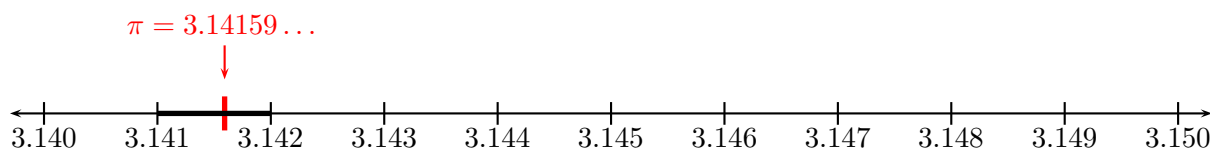
$\pi = 3.14159\ldots$

- π rounded to 2 decimal places is 3.14 (3.141...is closer to 3.14 than to 3.15). To see this, zoom in on the segment from 3.1 to 3.2 in the line above:
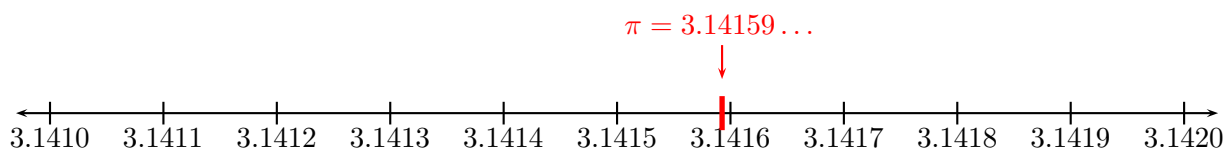
$\pi = 3.14159\ldots$

- π rounded to 3 decimal places is 3.142 (3.1415 is as close to 3.142 as to 3.141. So consider 3.14159..., this is closer to 3.142 than to 3.141). To see this, zoom in on the segment from

3.14 to 3.15 in the line above:

$$\pi = 3.14159\ldots$$

- π rounded to 4 decimal places is 3.1416 (3.14159... is closer to 3.1416 than to 3.1415). To see this, zoom in on the segment from 3.141 to 3.142 in the line above:

$$\pi = 3.14159\ldots$$

- 45.6 rounded to the nearest integer (whole number) is 46 (45.6 is closer to 46 than to 45).

- 7 651.23 rounded to the nearest integer is 7 651 (7 651.23 is closer to 7 651 than to 7 652).

Each time we had to decide what to do with the last digit kept (leave it as it is or increase it by 1). Mostly we just look at the following digit. If the following digit is greater than 5, increase the last digit. If the following digit is less than 5, leave the last digit as it is. If the following digit is 5 and there are more (non-zero) digits, increase the last digit. If the following digit is 5 and there are no more digits, we have a choice (we deal with this soon).

The same applies to negative numbers; we select the number with the correct number of digits which is closest to the original number.

Here are some more examples

- Round 10.3756 to 2 decimal places: 10.38 (as 10.3756 is closer to 10.38 then to 10.37);

- Round −10.3756 to 2 decimal places: −10.38 (as −10.3756 is closer to −10.38 than to −10.37);

- Round 9.999 to 1 decimal place: 10.0 (as 9.999 is closer to 10.0 than to 9.9);

- Round −9.999 to 1 decimal place: −10.0 (as −9.999 is closer to −10.0 than to −9.9);

In these examples we saw that the minus sign, "−", at the front of the negative numbers didn't change what is done to the digits. In general, rounding negative numbers is no different to rounding positive numbers.

- Round 9.15 to 1 decimal place: 9.1 and 9.2 are both possible answers as they are the same distance from 9.15.

Rounding a number like 9.15 to 1 decimal place can be done in several ways. There is no one correct answer. Some always round up (giving 9.2 in this case), some down (giving 9.1). Another option is to round up or down so as to make the last digit even. This is not the easiest method, but is by far the best for data collection (for example, measuring heights of people) as the average of the data collected will be more accurate. Using this method, 9.15 rounded to one decimal place is 9.2 (because 2 is even).

Using the method of rounding a 5 (which has no digits after it) up or down so as to make the last digit even we get

- 9.5 rounded to the nearest whole number (0 decimal places) is 10;

- 8.5 rounded to the nearest whole number (0 decimal places) is 8;

- 345.6565 rounded to 3 decimal places is 345.656;

- 345.6565 rounded to 1 decimal place is 345.7 (as it is closer to 345.7 than to 345.6).

- -9.5 rounded to the nearest whole number (0 decimal places) is -10;

- -8.5 rounded to the nearest whole number (0 decimal places) is -8;

Practice questions

11. Round 9.15 to the nearest whole number.

12. Round 234.876 to the nearest whole number.

13. Round 2.718281828459 to (a) the nearest whole number, (b) 1 decimal place, (c) 4 decimal places, (d) 7 decimal places.

14. Round -2.718281828459 to (a) the nearest whole number, (b) 1 decimal place, (c) 4 decimal places, (d) 7 decimal places.

15. Round -222.12345 to 4 decimal places.

4   More on rounding

Here some more examples of rounding.

- Round 6.817 to the nearest whole number.

  Answer: From the number line we can see that 6.817 is much closer to 7 than to 6. So the answer is 7.

  6.817

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  ←|————|————|————|————|————|————|————|————|————|————|————→
   6.0  6.1  6.2  6.3  6.4  6.5  6.6  6.7  6.8  6.9  7.0
  ```
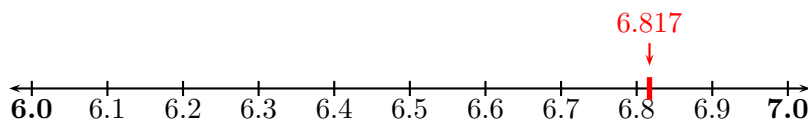
  Alternatively, the answer is either 6 or 7. To decide which, we look at the digit after the decimal point. It is the 8 in 6.**8**17. As 8 is greater than 5, we increase the 6 (the 6 in **6**.817) so it becomes 7. The answer is 7.

- Round 6.817 to 1 decimal place.

  Answer: From the number line we can see that 6.817 is closer to 6.8 than to 6.9, so the answer is 6.8.

- Round −6.817 to the nearest whole number.

  Answer: This is much the same as rounding 6.817 to the nearest integer. The answer is −7 because −6.817 is closer to −7 than to −6.

- Round 13.42 to the nearest whole number.

 Answer: It is 13 because 13.42 is closer to 13 than to 14.

  Alternatively, the answer is either 13 or 14. To decide which, we look at the digit after the decimal point. It is the 4 in 13.**4**2. As 4 is less than 5, we leave the 3 (the 3 in 1**3**.42) unchanged. The answer is 13.

- Round −13.42 to the nearest whole number.

  Answer: It is −13 because −13.42 is closer to −13 than to −14.

- Round 0.9765625 to (a) 1 decimal place, (b) 4 decimal places, (c) 6 decimal places.

  Answer: (a) 0.9765625 is between 0.9 and 1.0. It is closest to 1.0, as can be seen from the number line. So 1.0 is the answer.
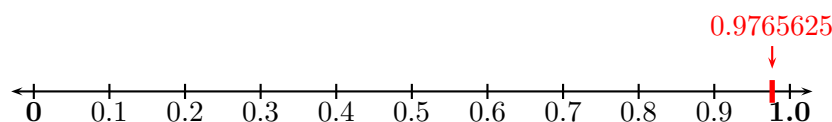
  Or, we can look at the digit after the 9 (the 9 in 0.**9**765625), which is 7. As 7 is greater than 5, we increase the 9. This means that we go up from 0.9 to 1.0.

  (b) The answer is either 0.9765 or 0.9766, but which one? As 0.9765625 is closest to 0.9766, 0.9766 is the answer.

  Or, find the digit in the 4th decimal place, this is 5 (the 5 in 0.976**5**625). Now find the digit following this, which is 6 (the 6 in 0.9765**6**25). As 6 is greater than 5, we increase 5 by 1 to get 0.9766.

  (c) The answer is 0.976562 or 0.976563. These are the same distance from 0.9765625 so either can be considered correct. If we want to round to make the last digit of the answer even, then we take 0.976562 as the answer. Or, find the digit in the 6th decimal place, this is 2 (the 2 in 0.97656**2**5). The 2 is followed by 5, and there is nothing after the 5. So we can take either 0.976562 or 0.976563 as the answer. If we want to round up or down to make the last digit of the answer even, then we take 0.976562 as the answer.

More practice questions for rounding

16. Round 1 026.3847 to the nearest integer.

17. Round −1 026.3847 to the nearest whole number.

18. Round 54.78 to 1 decimal place.

19. Round −54.78 to 1 decimal place.

20. Round 2.55 to (a) the nearest whole number, (b) 1 decimal place.

Mixed practice questions

21. Round 34.55 to the nearest integer.

22. Truncate 34.55 to give an integer.

23. Round 234.876 to the nearest integer.

24. Truncate 234.876 to give an integer.

25. Round 44.44 to 1 decimal place.

26. Truncate 44.44 to give a number 1 decimal place.

27. Round -45.678 to 2 decimal places.

28. Truncate -45.678 to give a number with 2 decimal places.

5   Answers to practice questions

1. 9.

2. 9.1.

3. 234.

4. 234.87.

5. −234.87.

6. Truncate 1 026.3847 to give an integer. 1 026.

7. Truncate −1 026.3847 to give a whole number. −1 026.

8. Truncate 54.78 to 1 decimal place. 54.7.

9. Truncate −54.78 to 1 decimal place. −54.7.

10. Truncate 2.55 to (a) give a whole number, (b) 1 decimal place. (a) 2 (b) 2.5.

11. 9.

12. 235.

13. (a) 3, (b) 2.7, (c) 2.7183, (d) 2.7182818.

14. (a) −3, (b) −2.7, (c) −2.7183, (d) −2.7182818.

15. −222.1234 or −222.1235 as they are both the same distance from −222.12345.

16. Round 1 026.3847 to the nearest integer. 1 026.

17. Round −1 026.3847 to the nearest whole number. −1 026.

18. Round 54.78 to 1 decimal place. 54.8.

19. Round −54.78 to 1 decimal place. −54.8.

20. Round 2.55 to (a) the nearest whole number, (b) 1 decimal place. (a) 3 (b) 2.6 (2.5 is also acceptable).

21. 35.

22. 34.

23. 235.

24. 234.

25. 44.4.

26. 44.4.

27. −45.68.

28. −45.67.