Solutions to Manipulating Relationships Problems

1. We are given the formula $d = \frac{m}{v}$ and the known variable are the density (d), 0.785 and the mass (m) which has a value of 10.
   As we are asked to find the volume (v) of the 10g sample of isopropyl alcohol we can rearrange the relationship $d = \frac{m}{v}$ to isolate v.

   $$d = \frac{m}{v}$$

   Multiply both sides by v gives

   $$dv = m$$

   Now, divide both sides by d to isolate v

   $$\begin{aligned} \frac{dv}{d} &= \frac{m}{d} \\ \frac{\not{d}v}{\not{d}} &= \frac{m}{d} \\ v &= \frac{m}{d} \end{aligned}$$

   On substituting the values $m = 10$ and $d = 0.785$ into the relationship $v = \frac{m}{d}$

   $$\begin{aligned} v &= \frac{10}{0.785} \\ &= 12.7388535 \end{aligned}$$

   That is, the volume of the 10g sample of isopropyl alcohol is approximately 12.74 mL.

2. Rearrange the relationship $t_C = \frac{5}{9}(t_F - 32)$ to isolate t_F on one side:

   $$t_C = \frac{5}{9}(t_F - 32)$$

   Multiply both sides by 9:

   $$\begin{aligned} 9 \times t_C &= 9 \times \frac{5}{9}(t_F - 32) \\ 9t_C &= 5(t_F - 32) \end{aligned}$$

Expanding the right hand side gives:

$$9t_C = 5t_F - 160$$

Now add 160 to both sides:

$$\begin{aligned} 9t_C + 160 &= 5t_F - 160 + 160 \\ 9t_C + 160 &= 5t_F \end{aligned}$$

And lastly, divide both sides by 5 to isolate t_F, that is:

$$\frac{9t_C + 160}{5} = t_F.$$

Now we can substitute $t_C = 30$ into the relation above, giving:

$$\begin{aligned} \frac{9 \times 30 + 160}{5} &= t_F \\ 86 &= t_F \end{aligned}$$

That is, 30 degrees Celsius corresponds to 86 degrees Fahrenheit.

3. In this question, we are asked to find the value for energy. From the relationship specific heat $= \frac{E}{mT}$ we need to isolate the variable E. We can do this by multiplying both sides by mT:

$$\begin{aligned} mT \times \text{specific heat} &= \frac{E}{mT} \times mT \\ mT \times \text{specific heat} &= \frac{E}{\cancel{mT}} \times \cancel{mT} \\ mT \times \text{specific heat} &= E. \end{aligned}$$

Now we can substitute the values given for specific heat, $4.184 J/g^oC$, mass (m), 250g and temperature change (T), 60^o (as the change in temperature is given by $80^o - 20^0$).
Therefore,

$$\begin{aligned} E &= mT \times \text{specific heat} \\ &= 250 \times 60 \times 4.184 \\ &= 62,760. \end{aligned}$$

That is, the heat energy required is 62,760 Joules.