



# Mathematics Advanced Student Workbook

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Student name: \_\_\_\_\_



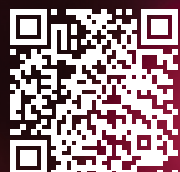


WESTERN SYDNEY  
UNIVERSITY



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**WESTERN**

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Just because you don't fit the usual entry-mould for uni doesn't mean you can't get in. Maybe you didn't get the ATAR you wanted. Maybe you don't have an ATAR at all. Maybe there were things happening in your life that got in the way of your study.

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# LOVE THESE SUBJECTS? SEE WHERE THEY CAN TAKE YOU! FIND YOUR PATH FROM HSC TO UNIVERSITY.

## YOU LOVE ENGLISH? *WHY NOT STUDY:*

- B Communication
- B Screen Media (Arts and Production)
- B Education (Primary)
- B Education (Secondary)
- B Arts
- B Arts (Interpreting and Translation)
- B Laws
- B Performing Arts (Acting)
- B Music
- B Design (Visual Communication)
- B Music
- B International Studies

## YOU LOVE STUDIES OF RELIGION? *WHY NOT STUDY:*

- B Arts
- B Social Science
- B Education (Primary)
- B Education (Secondary)

## YOU LOVE HISTORY? *WHY NOT STUDY:*

- B Arts
- B International Studies
- B Social Science
- B Education (Primary)
- B Education (Secondary)
- B Tourism and Event Management
- B Planning (Pathway to M Urban Management and Planning)

## YOU LOVE VISUAL ARTS? *WHY NOT STUDY:*

- B Architectural Design
- B of Design
- B of Design and Technology
- B of Industrial Design (Honours)

## YOU LOVE BUSINESS STUDIES? *WHY NOT STUDY:*

- B Business
- B Accounting
- B Education (Primary)
- B Education (Secondary)
- B Tourism and Event Management
- B Construction Management (Honours)

## YOU LOVE LEGAL STUDIES? *WHY NOT STUDY:*

- B Criminology
- B Criminal and Community Justice
- B Cyber Security and Behaviour
- B Policing
- B Laws
- B Social Science
- B Humanitarian and Development Studies
- B Education (Primary)
- B Education (Secondary)
- B Construction Management (Honours)
- B Social Work
- B Youth Work

## YOU LOVE COMMUNITY AND FAMILY STUDIES STUDIES? *WHY NOT STUDY:*

- B Arts
- B Criminology
- B Nursing
- B Midwifery
- B Psychology (Honours)
- B Psychological and Social Sciences
- B Social Science
- B Youth Work
- B Social Work
- B Humanitarian and Development Studies
- B Education (Early Childhood)
- B Education (Primary)
- B Education (Secondary)

## YOU LOVE PHYSICS? *WHY NOT STUDY:*

- B Science
- B Education (Primary)
- B Education (Secondary)
- B Engineering
- B Architectural Design
- B Industrial Design (Honours)
- B Design and Technology

## YOU LOVE CHEMISTRY? *WHY NOT STUDY:*

- B Science
- B Medical Science
- B Education (Primary)
- B Education (Secondary)

## YOU LOVE HEALTH AND MOVEMENT SCIENCE? *WHY NOT STUDY:*

- B Health Science
- B Health, Physical Education and Sport
- B Exercise Physiology
- B Exercise Science
- B Sport and Performance Science
- B Occupational Therapy
- B Paramedicine
- B Physiotherapy
- B Podiatric Medicine
- B Speech Pathology
- B Traditional Chinese Medicine
- B Nursing
- B Midwifery
- B Education (Primary)

## YOU LOVE MATHEMATICS? *WHY NOT STUDY:*

- B Computer Science
- B Data Science
- B Information and Communication Technology
- B Information Systems
- B Mathematics
- B Engineering
- B Education (Primary)
- B Education (Secondary)
- B Planning (Pathway to M Urban Management and Planning)
- B Accounting

## YOU LOVE BIOLOGY? *WHY NOT STUDY:*

- B Health Science
- B Health, Physical Education and Sport
- B Occupational Therapy
- B Paramedicine
- B Physiotherapy
- B Podiatric Medicine
- B Speech Pathology
- B Sport and Performance Science
- B Traditional Chinese Medicine
- B Science
- B Medical Science
- B Nursing
- B Midwifery
- B Education (Primary)
- B Education (Secondary)
- B Clinical Science/Doctor of Medicine

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## 2 Unit Advanced Mathematics Lecture

### Lecture Notes

#### Statistical Analysis – Probability

- 1 The discrete random variable  $X$  has probability distribution shown in the table below. It is known that  $E(X) = 0.55$

|            |    |   |     |      |      |
|------------|----|---|-----|------|------|
| $x$        | -1 | 0 | 1   | 2    | 3    |
| $P(X = x)$ | a  | b | 0.2 | 0.15 | 0.13 |

- (i) By forming a pair of simultaneous equations, or otherwise, find the values of a and b. 3
- (ii) Calculate  $\text{Var}(X)$ . 2

- 2 A pet ownership survey resulted in the following results:

$$P(C) = \frac{3}{7}, P(D | \bar{C}) = \frac{2}{5}, \text{ and } P(\bar{D} | C) = \frac{3}{4}$$

Where  $C$  is the event that "a person has a cat" and  $D$  is the event that "a person has a dog"

- (i) Complete the probability tree by marking a probability on each branch. 2
- (ii) If one person is chosen at random, find the probability that the person has:
- (a) a cat and a dog 1
- (b) at least one pet (cat or dog). 2

- 3 The probability distribution of a discrete random variable  $X$  is given by the table below.

|     |   |   |   |   |   |
|-----|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
|-----|---|---|---|---|---|

|            |     |          |     |         |     |
|------------|-----|----------|-----|---------|-----|
| $P(X = x)$ | 0.2 | $0.6p^2$ | 0.1 | $1 - p$ | 0.1 |
|------------|-----|----------|-----|---------|-----|

- (i) Show that  $p = \frac{2}{3}$  or  $p = 1$  2
- (ii) Let  $p = \frac{2}{3}$ , calculate to two decimal places  $E(X)$ ; and 2
- (iii)  $P(X \geq E(X))$  1

**Exponential function**

- 1 Find  $\int e^{1-3x} dx$  2
- 2 Find  $\int (e^{4x} + 1)^2 dx$  2
- 3 Differentiate  $y = \frac{e^{2x}}{2x+1}$ . (leave your answer in simplified form) 2
- 4 Differentiate  $e^{\cos \frac{x}{\pi}}$  2
- 5 Solve the following equation 2
- $$e^{2x} - 5e^x + 4 = 0$$
- 6 (i) Graph the curve  $y = e^x + 1$ , showing all essential features. 1
- (ii) Find the exact area bounded by the curve  $y = e^{2x} + 1$ , the  $x$ -axis,  $x = 0$  and  $x = \ln 2$ . 2

- 7 Find the coordinates of the point on the curve  $y = 2e^{3x} + 1$ , where the tangent to this curve is parallel to the line  $12x - y + 1 = 0$ . 3

**Logarithm function**

1      If  $f'(x) = \frac{2}{x}$  and  $f(e) = 1$ , find an expression for  $f(x)$ .      2

2      (i)      Show that  $6 + \frac{6x}{x^2 + 1} = \frac{6x^2 + 6x + 6}{x^2 + 1}$       1

(ii)       $\int_0^1 \frac{6x^2 + 6x + 6}{x^2 + 1} dx$       2

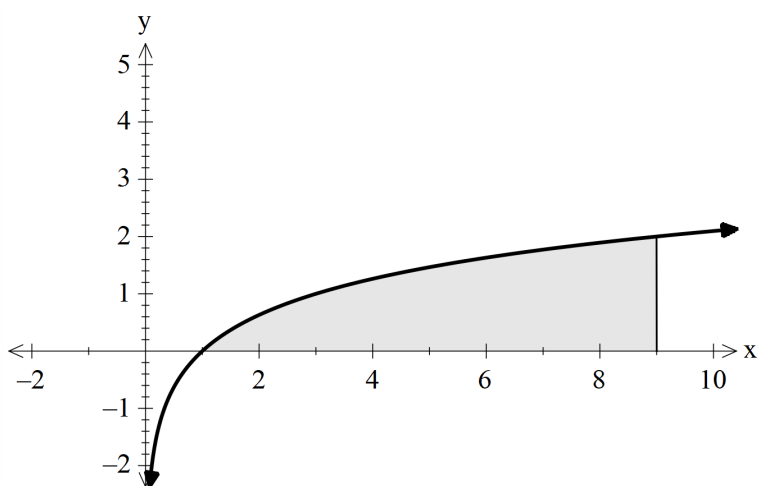
Hence find the exact value of

3      Differentiate  $(\ln x + 4)^5$ .      2

4      Find  $\int \frac{4x}{4x^2 + 3} dx$       2

Find

5      Find the equation of the tangent to the curve  $y = \ln(2x + 1)$ , at  $x = 0$ .      3

6            3

The diagram above shows the graph of  $y = \log_3 x$ . Find the area bounded by the curve, the line  $x = 9$  and the  $x$ -axis.

### Exponential equations

- 1 The current COVID-19 pandemic rate of infection in a town is given by 2  
 $\frac{dP}{dt} = 480e^{0.06t}$ ; where  $t$  is measured in days and  $P$  represents the number of people in the town at time  $t$ . Calculate the increase of in the number of people infected in the first 50 date.
  
- 2 The function  $C(t) = 10(e^{-t} - e^{-2t})$  is used to model the concentration  $C$  (in mg/L) at time  $t$  (in minutes) of a drug injected into the bloodstream.
  - (i) Find the concentration as  $t \rightarrow \infty$  1
  - (ii) Find an expression for  $C'(t)$ , the rate at which the drug is cleared from circulation 1
  - (iii) When is the rate equal to 0 2
  
- 3 A cup of tea was served to a customer at a restaurant. The temperature of the tea can be modelled by  $T = 21 + 69(2)^{-0.24t}$ , where  $T$  is the temperature of the tea, in degrees Celsius,  $t$  minutes after it is served.
  - (i) What is the temperature of the tea 10 minutes after it has been served? (2 decimal places) 1
  - (ii) At what rate is the tea cooling 10 minutes after it has been served? 2
  - (iii) How long after the tea is served will it take for its temperature to reach  $31^\circ C$  3

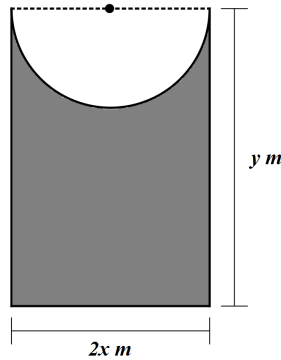
- 3 The blood-alcohol content ( $A$ ) after a person has been drinking is given by  $A = A_0 e^{-kt}$ , where  $A_0$  represents the blood-alcohol content level at the time a person stops drinking,  $t$  is measured in *hours* and  $A$  in *mg/ml*.

Melita stops drinking at 11pm on Saturday night (  $t = 0$  ) and her blood alcohol level was measured at  $0.24 \text{ mg/ml}$ . It took 28 hours for Melita's blood-alcohol level to be measured at  $0.001 \text{ mg/ml}$ .

- (i) Explain why  $A_0 = 0.24$  and find the value of  $k$ . (4 decimal places) 2
- (ii) The allowable blood-alcohol level limit for Melita to drive a car is  $0.05 \text{ mg/ml}$ . What is the earliest time on Sunday that Melita will be able to legally drive? (leave your answer to the nearest hour) 2
- (iii) What is the rate of decrease of the blood-alcohol level content in Melita's blood at 8.00am on Sunday? 1

## Maxima/Minima - Optimisation

1

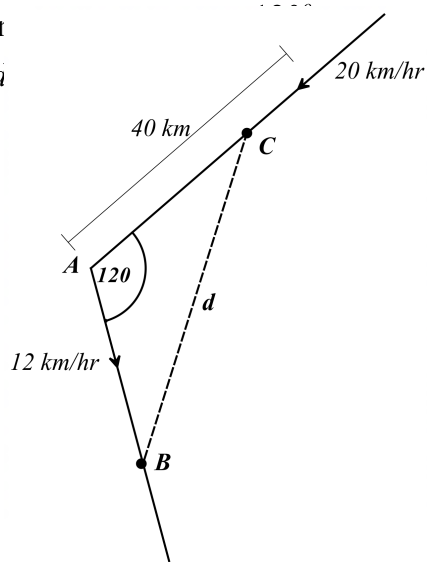


A farmer wishes to fence off an enclosure in the above shape (*3 straight lines and a semi-circle*) so that the area ( $A$ ) enclosed is equal to  $200 \text{ m}^2$ . The width and length are  $2x \text{ cm}$  and  $y \text{ cm}$  respectively.

- (i) Show that  $y = \frac{100}{x} + \frac{\pi x}{4}$ . 2
- (ii) Find the value of  $x$  so that the farmer uses the least amount of fencing to form this enclosure. 3

2

Two straight roads meet at  $A$ . At time  $t = 0$ , marathon runner Bassam ( $B$ ) leaves point  $A$  and runs at  $12 \text{ km/hr}$  on one road. At the same time, Ceren ( $C$ ) is  $40 \text{ km}$  away from  $A$  on the other road and rides his bike towards  $A$  at  $20 \text{ km/hr}$ . The two roads meet at an angle of  $120^\circ$ . The distance between  $B$  and  $C$  at time  $t$  hours is  $d$ .



(i) Show that  $d^2 = 304t^2 - 1120t + 1600$ . 3

(ii) Find the minimum distance between Bassam and Ceren (2 decimal places). 3

## Trigonometry - Calculus

1 (i)  $(\sin x + 1)^3$  . 2

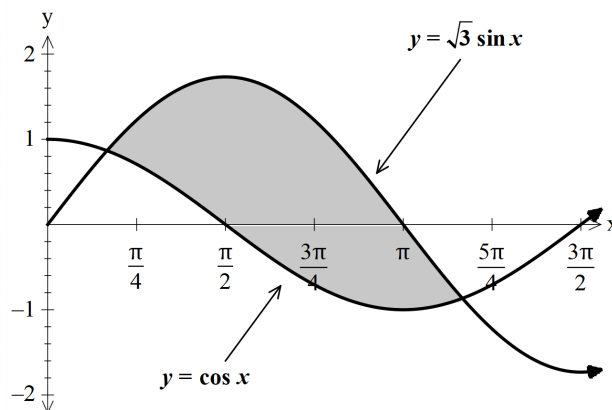
(ii)  $y = x \cos 2x$  . 2

(iii)  $y = \tan^2(5x + 1)$  , leaving your answer in terms of  $\tan$  only. 2

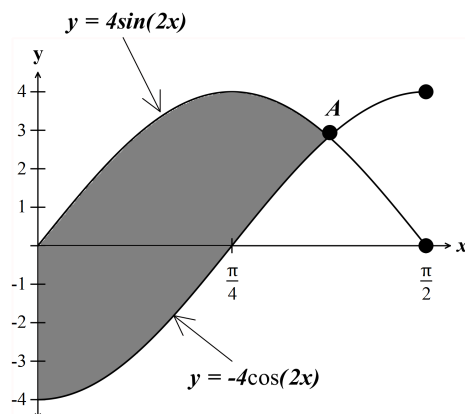
2  $\int_{\frac{\pi}{16}}^{\frac{\pi}{12}} \tan^2 4x \, dx$  2

Given that  $\sec^2 x = 1 + \tan^2 x$  ; evaluate

3 In the diagram below, the curves  $y = \sqrt{3} \sin x$  and  $y = \cos x$  . Find the area of the shaded region.



In the diagram below, the curves  $y = 4 \sin 2x$  and  $y = -4 \cos 2x$ ,  $0 \leq x \leq \frac{\pi}{2}$  meet at  $A$ .



- (i) By equating the two equations  $y = 4 \sin 2x$  and  $y = -4 \cos 2x$ ; and 2  
solving simultaneously, show that the  $x$ -coordinate of  $A$  is  $x = \frac{3\pi}{8}$ .  
(ii) Hence, find the exact area of the shaded region. 2  
)

### Rates of change - Motion

1 The displacement  $x(t)$  of a particle moving in a straight line at time  $t$  is  $x(t) = t^3 - 4.5t^2 - 12t$ , where  $t$  is measured in seconds and  $x$  in metres

- (i) Find an expression for the velocity at time  $t$  1
- (ii) Find the time(s) when the particle comes to rest. 2
- (iii) Find the total distance travelled in the first 10 seconds 2

2 The acceleration of a particle is given by

$a = 8e^{-2t} + 3e^{-t}$  where  $t$  (time) is measured in seconds. Initially, its velocity is  $-6\text{ms}^{-1}$  and its displacement is  $5\text{ m}$ .

- (i) Show that the displacement of the particle is given by: 2

$$x = 2e^{-2t} + 3e^{-t} + t$$

- (ii) Find the time when the particle comes to rest. 2
- (iii) Find the displacement when the particle comes to rest. 1

3 A particle is moving in a straight line. Its displacement  $x$  metres from a fixed point  $O$  is given by:

$$x = 2 \sin 2t - 2t \quad \text{where } t \text{ is measured in seconds.}$$

- (i) Find an expression for the velocity and acceleration of the particle in terms of  $t$ . 2

- (ii) Find the first two occasions when the particle is at rest after  $t = 0$  . **2**
- (iii) Find the distance travelled between these two occasions, leaving your answer in exact form. **2**
- (iv) Find the maximum acceleration of the particle. **1**

### Investment, annuities, and loans

- |   |                                                                                                                                                                                                                              |   |
|---|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|
| 1 | Ricky invested \$2400 at 6.5% p.a. If the future value of this investment amounted to \$3237; find the number of years and months this amount was invested for.                                                              | 2 |
| 2 | (i) An amount of \$80 per month is invested into a bank account at the end of each month. The account pays 9% p.a. interest paid monthly on investments. Find the future value of this annuity at the end of 12 years.       | 3 |
|   | (ii) Find the present value of the total pay-out achieved in part (i)                                                                                                                                                        | 1 |
| 3 | Felix bought a second hand car for \$30,000 with borrowed money from a finance company which charged him 18% p.a. reducible interest, reducible monthly. Felix agreed to pay back the loan plus interest at \$900 per month. |   |
|   | (i) Show that the amount that Felix owed after his second payment was<br>$A_2 = 30,000(1.015)^2 - 900(1+1.015)$                                                                                                              | 1 |
|   | (ii) Show that the amount owing after $n$ payments have been made can be expressed as $A_n = 60,000 - 30,000(1.015)^n$                                                                                                       | 3 |
|   | (iii) Hence find the number of months that Felix required to pay back the loan plus interest.                                                                                                                                | 1 |



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